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MODELLING OF POTENTIALLY HAZARDOUS OBJECTS WITH DEVIATING ARGUMENTS AND THEIR CONTROL

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In the paper the mathematical modelling and simulation of the potentially hazardous object (PHO) is considered as a statistical non-linear dynamical system with deviating arguments. Such kind of deviating arguments may be different kind time shifts (time delays and time forecasting terms), which are very important with regards to real peculiarities of the PHO. The vital problem of a development of the mathematical models and simulation of the potentially hazardous objects was considered in touch with a need of a tactical and strategic planning and decision making support in a lot of the environmental and other tasks. The reason for an importance of such models and computer simulations with their use was connected to prediction of some negative environmental influence of the PHO. Development of the aggregate PHO models provided the possibilities to systematic investigation of the crucial situations in their evolution and peculiarities in control parameters, bifurcation of the regimes, catastrophes, etc. Certainly the results obtained have had concern to any other dynamical systems with deviating arguments, which have specific peculiarities of their modelling and control due to their arguments.

1 Problem formulation

Many potentially hazardous objects could be considered as non-linear statistical dynamical system that consists of a number of different parameters, which are interconnected. As far as any PHO is a too complicated dynamical system to build its deterministic model, it seemed to be quite reasonable idea to construct (based on the available statistical data) its aggregate model. In general, the aggregate model of PHO would be represented as follows:

$$\frac{dx_i}{dt} = (N_i - x_i)\cdot(a_{i0} + \sum_{j=0}^{n} a_{ij}\cdot(N_j - x_j)), \qquad (1)$$

where are: x_i - PHO characteristic parameters, N_i - limit values of *i*-th parameter, a_{ij} - coefficients, *n*-number of parameters, *i=1,2..n.* Strictly speaking, in general case a_{ij} should be some functions of time; N_i may vary according to the problem being stated (tactical or strategic planning). The development rate of each variable in the system (1) was determined by other variables and decreased with reaching the limits being stated (saturation property) [1]. The set of the non-linear ordinary differential equations (1) with corresponding initial data allowed simulating the PHO evolution in time until the achievement of the planning values or any kind of other regimes, e.g. some saturation, critical point, catastrophe, etc.

The parameters x_i could represent for instance the number of personnel and managers, the amount of the product output, expenditures required for elimination of the negative influence on the environment, the PHO repair and restore expenses, the level of safety culture, etc. The variables may change in the range from 0 to N_i (N_0=1 is chosen just for simplicity).The first step of PDO model development consists in calculation of the constants in the equation array (1) using the statistical data available. Unfortunately for a lot of PDO this is crucial problem due to an absence or uncertainty of the statistical data about its functioning. The equation array (1) could be treated under some special conditions and restrictions due to real peculiarities of the PHO. Therefore further consideration of PHO modelling could be represented as the problem of optimal control by parameters a_{ij}(t). The PHO would be going to the regime being stated by the stated criteria or by a couple of criteria: e.g., the minimal time for reaching of the regime stated, the maximum of the product output, the minimum of the environmental damage required, etc. The analysis of the system (1) showed two types of the stationary solutions available:

$$x_i = N_i , \qquad x_i = x_i^* , \tag{2}$$

where x_i^* were the solutions of the following algebraic equations:

$$a_{i0} + \sum_{j=0}^{n} a_{ij} \cdot (N_j - x_j) = 0. \tag{3}$$

If determinant of the system (3) was nonzero, one could obtain a single solution $x_i = x_i^*$. If the determinant was zero, there were possible the following cases: 1) nonzero (n-1)-th order minors and there are $x_i = x_i^*$ - (n-1) direct lines depending of one parameter (one of the variables x_i); 2) nonzero (n-k)-th order minors, then there is k-th order surface $x_i = x_i^*$ in n-dimensional parametric space. The coefficients a_{ij} of the system (1) should be computed for the real data from a simulated object functioning. This is a problem of the regression analysis. One can determine a_{ij} for instance using the condition of a minimal mean-square deviation of the solution x_i(t) from the real data $\tilde{x}_i$(t). The analysis of singular points for the system (1) is important for a problem of the PHO control. Consider for example the model of a nuclear power industry. It was assumed that the fundamental variables of a considered system are the following: x_1, x_2 - the numbers of the personnel and managers, respectively; x_3 - the total product output; x_4 - the PHO repair and restore expenditures; x_5 - the expenses required for elimination of a negative influence on the environment; x_6 - the safety culture level. Based on the experience one could define possible interconnections among the parameters in the system in accordance with the following description. Assuming that the safety culture level may vary in the range [0, 1], where x_6=0 corresponds to an absolutely impossible in practice case of safety culture absence, one could suggest that x_6=1 corresponds to an ideal case, when a further growth of safety culture is impossible.

2 The model

If all variables of the model were divided by their limit values, then taking into account all interconnections of the PHO parameters determined from practical knowledge, the equation array (1) might be got in the following dimensionless form:

$$\begin{aligned}
\frac{dy_1}{dt} &= [b_{10} + b_{11}(1-y_1) + b_{12}(1-y_2) + b_{13}(1-y_3)](1-y_1), \\
\frac{dy_2}{dt} &= [b_{20} + b_{21}(1-y_1) + b_{22}(1-y_2) + b_{23}(1-y_3)](1-y_2), \\
\frac{dy_3}{dt} &= [b_{30} + b_{31}(1-y_1) + b_{32}(1-y_2) + b_{33}(1-y_3) + b_{34}(1-y_4) + b_{35}(1-y_5) + b_{36}y_6)](1-y_3), \\
\frac{dy_4}{dt} &= [b_{40} + b_{43}(1-y_3) + b_{44}(1-y_4) + b_{45}(1-y_5) + b_{46}y_6)](1-y_4), \\
\frac{dy_5}{dt} &= [b_{50} + b_{53}(1-y_3) + b_{54}(1-y_4) + b_{55}(1-y_5) + b_{56}y_6)](1-y_5), \\
\frac{dy_6}{dt} &= [b_{60} + b_{61}(1-y_1) + b_{62}(1-y_2) + b_{63}(1-y_3)](1-y_6),
\end{aligned} \tag{4}$$

where $y_i = x_i / N_i , b_{ij} = a_{ij} N_j , b_{i0} = a_{i0} , y_6 = x_6$ were introduced. Here the restriction $N_4 + N_5 \le N_3$ was assumed. For the dimensionless time t some characteristic interval T was taken.

An indicator for the described PHO safety level was proposed as follows:

$$V = q \frac{1 + \alpha_1 y_1 + \beta_1 y_2}{1 + \alpha_2 y_1 + \beta_2 y_2} \cdot \frac{y_3 y_4 y_6}{y_5} , \tag{5}$$

where q is the parameter that characterizes a current level of a society production (an engineering level), α_i , β_i - coefficients.

The proposed relation (5) shows that by $y_3, y_4, y_6 \to 0$, there is $V \to 0$, and by $y_5 \to 0$ there is $V \to \infty$. Whether $y_1, y_2 \to 0$, the overall PHO safety level indicator is not only determined by the number of the personnel but also by the rate of both the individual and society interests. Then some new variables were introduced $z_i = 1 - y_i$ simplifying the system (4). By certain conditions, some delays in a system might be available, e.g. delays connected with a speed of an information transfer or with a processing time. In those cases, one should introduce time delay for each of the parameters. In turn, the time delays τ could be in general also some functions of time. In a first approach, the time delays τ were supposed to be constant (at least for a short time). Then with account of the above-mentioned, the above-described PHO model was got as follows:

$$
\begin{aligned}
\frac{dz_1}{dt} &= [b_{10} + b_{11}z_1(t-\tau_{11}) + b_{12}z_2(t-\tau_{12}) + b_{13}z_3(t-\tau_{13})]z_1(t-\tau_{10}),\\
\frac{dz_2}{dt} &= [b_{20} + b_{21}z_1(t-\tau_{21}) + b_{22}z_2(t-\tau_{22}) + b_{23}z_3(t-\tau_{23})]z_2(t-\tau_{20}),\\
\frac{dz_3}{dt} &= [b_{30} + b_{31}z_1(t-\tau_{31}) + b_{32}z_2(t-\tau_{32}) + b_{33}z_3(t-\tau_{33}) + b_{34}z_4(t-\tau_{34}) + b_{35}z_5(t-\tau_{35}) + b_{36}z_6(t-\tau_{36})]z_3(t-\tau_{30}),\\
\frac{dz_4}{dt} &= [b_{40} + b_{43}z_3(t-\tau_{43}) + b_{44}z_4(t-\tau_{44}) + b_{45}z_5(t-\tau_{45}) + b_{46}z_6(t-\tau_{46})]z_4(t-\tau_{40}),\\
\frac{dz_5}{dt} &= [b_{50} + b_{53}z_3(t-\tau_{53}) + b_{54}z_4(t-\tau_{54}) + b_{55}z_5(t-\tau_{55}) + b_{56}z_6(t-\tau_{56})]z_5(t-\tau_{50}),\\
\frac{dz_6}{dt} &= [b_{60} + b_{61}z_1(t-\tau_{61}) + b_{62}z_2(t-\tau_{62}) + b_{63}z_3(t-\tau_{63}))]z_6(t-\tau_{60}),
\end{aligned}
\tag{6}
$$

where τ_{ij}=const. The mathematical model (6) thus obtained describes a PHO evolution in time influenced upon its history. In a similar way the positive time shifts can be introduced to the system (6) with respect to the processes of some planning levels for the future parameters' levels (e.g. managers orient in their activity to the desired level of some parameters). This system can be rewritten in the withinity of t, by use of a Taylor series for any function $z_i(t-\tau_{ij})$. Taking into account the Elsgoltz's theorem [2] one should keep in an expansion only the first-order terms over τ_{ij}: $z_i(t-\tau_{ij}) \approx z_i(t) - \tau_{ij}\dot{z}_i$, ($\dot{z} \equiv dz/dt$) because the terms neglected worse the fidelity. Thus, all the variables in the approximated system are simultaneous functions of a single time t.

3 An optimal control of the PHO

One could consider the problem of an optimal control of the PHO for the criterion stated. Then the Hamiltonian for the system (5) could be written [3]: $H(z,p) = \sum_{i=1}^{6} p_i z_i \sum_{j=0}^{6} b_{ij} z_j$, (7)

where $z_0 = 1, p_i$ were determined by the following equation array: $\frac{dp_i}{dt} = -(p_i \sum_{j=0}^{6} b_{ij} z_j + \sum_{j=1}^{6} p_j z_j b_{ji})$ (8)

Thus, the problem of the optimal control for PHO consists in the calculation of the parameters $u_m(t)$ (m=1,2,..M) from the following equations:

$$\frac{dz_i}{dt} = f_i(z_1,..z_6,u_1,..u_6), \tag{9}$$

where $u_i(t)$ could be treated as some additive control functions (discrete in time) or $b_{ij}(t)$. The quality functional may be stated as follows: $I = \int_0^T f_0(z_i,u_m)dt$, (10)

or

$$\dot{z}_0 = f_0(z_i,u_m), \quad z_0(0) = 0. \tag{11}$$

The optimal control is realized by $u_m(t)$ satisfying the equation (10) or (11) with the corresponding equation $H(z_i,p_i,u_m) = 0$. The Pontriagin's maximum principle presents only the necessary condition for the system's optimal control (but not the sufficient one). Therefore a number of different solutions can exist or a solution can be absent at all. Based on the mathematical models developed, two computer codes were developed according to the considered models with time shifts and without them. They were used for the simulation of the regimes of PHO evolution in time and study of its optimal control. Unfortunately the work with the validation of the model against real PHO situations is still in the process being very cumbersome from different points of view. Therefore a numerical simulation was done for the hypothetical object with some properties close to the real one just from the practical experience point of view. The results of numerical simulations performed by the model obtained have evidently shown that by some parameters dynamical system may have critical regimes. Such critical regimes may be got in case of high-frequent oscillations on phase portrait of the system. The results presented here correspond to a modeling data due to some obstacles in getting the real data, which have been collected and processed for the model during the last few years but still not finished.

4 Conclusion

The results obtained maybe of interest for many practical applications [4, 5] where a similar non-linear dynamical model may be got by aggregate generalized statistical data. Numerical simulation with the models allows understanding more deeply basic properties of the complex system, as well as investigates critical regimes available in the system under certain conditions. Decision-making group to make right choice in their strategic and tactical planning of the systems' development and control must discuss the results of modeling together with investigators. This will help the investigators and decision-making persons to understand it better. The following problems were met here. The first one has concern to the initial statistical data, which contain many uncertainties and are often difficult to prove. The other problem is in touch with modeling of a non-linear dynamical system, which has some deviating arguments in reality such as time delay by some parameters. It may happen due to a real time delay from any decision made by managers to its practical implementation, due to a finite speed of the information transfer in a real system, etc. Then it may happen due to time forecasting terms, for example, managers are often making decisions based on desired parameters to be achieved in the future instead of current real parameters. The above-mentioned features and problems cause the third problem of mathematical nature, which consists in solution of the non-linear system of differential equations with time shifts [6, 7], both negative (time delays) and positive (time forecast, or advanced terms, and so on). Having a number of negative and positive deviating terms in the equation array, one needs to develop some special methods for their solution. This problem stated here is presently under elaboration.

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