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PARAMETRIC CONTROL IN FLUID MECHANICS AS THE EFFECTIVE STABILIZING

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Parametric excitation of oscillations is done in some system by temporal variation of one or several parameters of a system (mass, momentum of inertia, temperature, stiffness coefficient; for the fluids: pressure, viscosity, etc.). Thus, parametric oscillations are excited and maintained by parametric excitation. Examples of parametric oscillations are oscillations of a stiffness coefficient due to the temperature variation in a loaded elastic solid, which are able to evoke later on its vibrations. Then oscillations of temperature (pressure) in fluid (gas) flow are able to evoke oscillations of its pressure (temperature) or (and) viscosity with the consecutive oscillations of other flow parameters. Electric and magnetic fields may cause the oscillations in flow of conductive fluid producing the oscillations of other parameters, etc.

One can consider a problem of the jet and film flows fragmentation and drop formation, which is of great importance for a lot of the modern industry and technology tasks: metallurgy, chemical technology, energy, etc. Because of complexity of the real physical systems there are also considered such processes together with the other ones, for example, phase crystallization and flows through porous (granular) media. The studying of such complicated problems came true only in the last decades due to computer science and mathematical simulation achievements. Now the problem is not only to simulate the processes but also to control them with regards the task being stated. Thus, the problem of the parametric control in fluid mechanics and in continua, in general, is important for the practical applications. The following three aspects can be mentioned by this:

- Excitation of the parameters' oscillations in continuous media in touch with the necessity of various technological and technical processes intensification: heat and mass transfer, mixing, decreasing of viscosity, improving of the crystallizating metal quality and many other phenomena.
- Excitation of the parameters' oscillations with regards the necessity of jet and film flows disintegration: air spray, spray-coating, metal spraying, desperation and granulation of materials (e.g. particles producing from the molten metals), etc.
- Suppression of the parameters' oscillations (stabilization of some unstable regimes and processes): the jet-drop and film screens designed for the protection of diaphragm of the experimental thermonuclear reactor, thermal instability and control of the fusion reactor, control of the electromechanical and electrochemical instabilities, the plasma and combustion stability, decreasing of the hydrodynamic and acoustic resistance, etc. In some cases the parametric control makes possible not only the processes' intensification but, what is more important, an operation of the processes, which are impossible without parametric control [1-3].

Mathematical model of the system

In controlling a phase transition boundary with the aid of an automatic heat flux regulator, the parameters of the regulating system's effects are programmed at the boundary line with the regulator. The latter is then considered to be hooked onto a powerful energy source, so that here the reverse effects of the object can be ignored. As for the solidified front, it is supposed to be a surface having constant temperature, whereas the phase transition stage is allegedly "zero thin". In other words, the transition from the liquid to the solid phase is a "leaping" process occurring at the phase transition boundary lines. In using heat flux regulators, considering that the perturbation boundaries of the solidified front lead to disturbances in the magnetic field, with the concurrent alterations in the winding electric current which, amplified in the thin skin-layer close to the interface, and which suppresses or reinforces the relevant perturbation by Joule heat release, then we arrive at the only logic conclusion:

$$dT_{m,k} / dn = G_{m,k} T_{m,k}, \qquad (1)$$

where *m, k* is the value denoting the harmonic number (wave number as per circumferential and longitudinal coordinates); T is the temperature; n is the phase distribution surface normal vector; $G_{m,k}$ is the control system's feed-back factor. The $G_{m,k}$ value may vary on a large scope, in that it can be altered constructively, so that it is possible to select a value of the factor $G_{m,k}$ for energy harmonic reading, necessary in solving the given problem. In an axial symmetry, the mathematical model of a system for perturbations in a linear approach is $div\,\vec{v}_1 = 0$ **,**

$$\partial\vec{v}_1 / \partial t + u_0 \partial\vec{v}_1 / \partial x = -1/\rho \nabla p_1, \qquad (2)$$

$$\rho_j c_j [\partial\tau_1 / \partial t + (2-j)(u_0 \partial\tau_1 / \partial x + \vec{v}_1 \nabla T_1)] =$$
$$\kappa_j (\partial^2 \tau_j / \partial r^2 + 1/r \partial\tau_j / \partial r + 1/r^2 \partial^2 \tau_j / \partial\varphi^2$$
$$+ \partial^2 \tau_j / \partial x^2),$$

where j=1,2, $\vec{v}_1 = \{u_1, v_1, w_1\}(r) u_0 \exp i(kx + m\varphi - \omega t)$, p_1, τ_1- perturbations of the velocity, pressure and temperature of a fluid. Index 2 corresponds to the parameters of a solid phase. The perturbation of a boundary of the solidified front is modeled as $r=R[1+\zeta \exp i(kx+m\varphi-\omega t)]$. Here T is the temperature of undisturbed system; ρ_j, c_j are density and specific heat of j-th phase. Boundary conditions are stated as:

1) from the symmetry assumption: $r=0$, $u_1=0$, $\tau_1=0$; (2)

2) on the perturbed boundary of solidified front $r=R[1+ \zeta \exp i(kx+m\varphi-\omega t)]$:

$$\tau_j(R,\varphi,x)+R\zeta(\partial T_j/\partial r)_{r=R}\exp i(kx+m\varphi-\omega t)=0, \ j=1,2 \qquad (3)$$

$$r=R, \quad u_1=(1-\rho_{2/1})\partial r/\partial t, \qquad \kappa_2\partial\tau_2/\partial r=\kappa_1\partial\tau_1/\partial r+\rho_2\lambda_{21}\partial r/\partial t; \qquad (4)$$

where $\rho_{2/1}=\rho_2/\rho_1$, $\partial r/\partial t$ is velocity of solidification front (movement of the boundary due to solidification-melting on it), λ_{21} is the heat of phase change (solidification);

3) on the wall surface ($r=R+r_0$), the impedance condition (1) is stated.

The general case of axisymmetrical film flow is described with the following dimensionless system of equations: $\partial\zeta/\partial t+1/r\partial\zeta/\partial r+1/r\partial q/\partial r=0$,

$$1/r\partial q/\partial t+\partial/\partial r(q/r^2)+\zeta/2\partial\zeta/\partial t[\alpha_u+\partial/\partial r(\partial\zeta/\partial t+1/r\partial\zeta/\partial r)]+\zeta\partial\zeta/\partial r(1/\mathbf{Fr}^2-$$

$$\mathbf{Eu}_g\cos\Omega t)+2\zeta\mathbf{Al}h_m\partial h_m/\partial r+1/\mathbf{Re}[3\alpha_u+4\partial/\partial r(\partial\zeta/\partial t+1/r\partial\zeta/\partial r)]-\zeta/\mathbf{We}\partial K_c/\partial r=0, \qquad (5)$$

$$\partial h/\partial t+1/r\partial h/\partial r=1/\mathbf{Re}_m(\partial^2 h/\partial r^2+1/r^2\partial^2 h/\partial\varphi^2+1/r\partial h/\partial r), \qquad (6)$$

where are: $K_c=\{\partial^2\zeta/\partial r^2+1/r\partial\zeta/\partial r[1+(\partial\zeta/\partial r)^2]\}/[1+(\partial\zeta/\partial r)^2]^{3/2}$, $q=\int_0^{\varsigma} urdz$, $\alpha_u=(\partial u/\partial z)_{z=0}$,

$\mathbf{Eu}_g=g_v a/u^2_0$- vibrating Euler's number, a- thickness of undisturbed film, u_0- velocity of vertical jet (characteristic velocity of a film flow), ζ- dimensionless perturbation of the film surface, $\mathbf{Fr}=ga/u^2_0$ Froude's number, $\mathbf{We}=au^2_0/\sigma$ - Weber's number, σ- surface tightness coefficient, $\mathbf{Al}=\mu_m h^2_m/(\rho_1 u^2_0)$- Alphven's number, $\mathbf{Re}=u_0 a/\nu$- Reynolds's number, $\mathbf{Re}_m=u_0 a/\nu_m$ - magnetic Reynolds's number. The equation array (5) was obtained by the integration of the film flow equations with correspondent boundary conditions across the film. In case of vibrating wave excitation in film flow the approximate solution of equation array (5) was obtained following:

$$\zeta=B\exp(r-1+\mathbf{Eu}_g/\Omega\sin\Omega t-t/\mathbf{Fr}^2), \qquad (7)$$

where B is a constant that was determined by the experiments.

In a flat case, a similar to the equation array (8), (9) dimensionless equation array was:

$$\partial q/\partial t+\partial q/\partial x+0{,}5(\partial^2\zeta/\partial t\partial x+\partial^2\zeta/\partial x^2-\alpha_u)\zeta\partial\zeta/\partial t+\zeta/\mathbf{Fr}^2\partial\zeta/\partial x+3/\mathbf{Re}(\partial^2\zeta/\partial t\partial x+\partial^2\zeta/\partial x^2-\alpha_u)+2\mathbf{Al}\bullet$$

$$\bullet\,\zeta h\partial h/\partial x-\zeta/\mathbf{We}\partial/\partial x\{\partial^2\zeta/\partial x^2[1+(\partial\zeta/\partial x)^2]^{-1.5}\}=0,$$

$\partial\zeta/\partial t+\partial\zeta/\partial x+\partial q/\partial x=0,$ (8)
$\partial h/\partial t+\partial h/\partial x=1/\mathbf{Re}_m\partial^2 h/\partial x^2;$

where $q=\int_0^{\varsigma} udz$ is. The equation array (8) was solved by the reductive perturbation method. First it was reducted to the following matrix form: $\partial\mathbf{U}/\partial t+\mathbf{A}(\mathbf{U})\partial\mathbf{U}/\partial x+\mathbf{C}(\mathbf{U})\partial^2\mathbf{U}/\partial x^2+\mathbf{B}(\mathbf{U})=0$, (9) where: $\mathbf{U}=[h, q, \zeta, \partial h/\partial t, \partial q/\partial t, \partial\zeta/\partial t, \partial h/\partial x, \partial q/\partial x, \partial\zeta/\partial x, \partial^2\zeta/\partial x\partial t, \partial^2\zeta/\partial x^2]^T$, and the matrices **A**, **C** are 11 by 11. The nonzero elements of the vectors and matrices are the following ones:
$b_1=-\partial h/\partial t$, $b_2=-\partial q/\partial t$, $b_3=-\partial\zeta/\partial t$,
$b_5=\alpha_u/2[\partial\zeta/\partial t(\partial q/\partial x+\partial\zeta/\partial x)+\zeta\partial^2\zeta/\partial x\partial t+2\mathbf{Al}\partial h/\partial x\bullet(\partial^2 h/\partial t\partial x+\partial^2 h/\partial x\partial t)+(\zeta\partial^2\zeta/\partial x\partial t+\partial\zeta/\partial x\partial\zeta/\partial t)/\mathbf{Fr}^2+3(\partial^2\zeta/\partial x^2)^2\{[1+(\partial\zeta/\partial x)^2]^{-5/2}/\mathbf{We}\}\{\partial\zeta/\partial t\partial\zeta/\partial x+\zeta\partial^2\zeta/\partial x\partial t-5\zeta\partial^2\zeta/\partial x\partial t(\partial\zeta/\partial x)^2/[1+(\partial\zeta/\partial x)^2]\}$;
$b_9=-\partial^2\zeta/\partial x\partial t$; $a_{4,1}=1$; $a_{5,4}=2\mathbf{Al}\partial h/\partial x$; $a_{5,5}=0,5\zeta(\alpha_u-\partial^2\zeta/\partial x\partial t-\partial^2\zeta/\partial x^2)+1$; $c_{4,4}=-1/\mathbf{Re}_m$;
$a_{5,8}=0,5[\partial\zeta/\partial t(\partial q/\partial x+\partial\zeta/\partial x)+\zeta\partial^2\zeta/\partial x\partial t]$; $a_{6,5}=a_{6,6}=1$; $a_{7,4}=a_{8,5}=a_{11,10}=-1$; $a_{5,10}=6\zeta\partial\zeta/\partial x\partial^2\zeta/\partial x^2[1+(\partial\zeta/\partial x)^2]^{-5/2}/\mathbf{We}$; $a_{5,11}=\{3\zeta(\partial\zeta/\partial x\partial^2\zeta/\partial x\partial t-\partial\zeta/\partial t[1+(\partial\zeta/\partial x)^2]\}[1+(\partial\zeta/\partial x)^2]^{-5/2}/\mathbf{We}$; $c_{5,5}=0,5\zeta(\partial q/\partial x+\partial\zeta/\partial x)-3/\mathbf{Re}$; $c_{5,10}=-\zeta[1+(\partial\zeta/\partial x)^2]^{-3/2}/\mathbf{We}$; $c_{10,5}=c_{10,6}=1$.
The solution of the obtained standard evolutionary equation (8) was found in the form [2, 3]:
$\mathbf{U}=\sum_{\alpha=0}^{\infty}\varepsilon^{\alpha}\mathbf{U}^{(\alpha)}$, where are: $\varepsilon=o(1)$, $\mathbf{U}^{(\alpha)}=\sum_{l=-\infty}^{+\infty}\mathbf{U}_l^{(\alpha)}(\xi,\tau)\exp il(kx-\omega t)$; $\xi=\varepsilon(x-v_g t)$; $\alpha\geq 1$; $\tau=\varepsilon^2(t)$;
$\mathbf{U}_1^{(1)}=\mathbf{R}\varphi$; $\mathbf{W}_1\mathbf{R}=0$; $\mathbf{L}\mathbf{W}_1=0$; $\mathbf{W}_1=||-il\omega\mathbf{I}+ilk\mathbf{A}^{(0)}+\nabla\mathbf{B}^{(0)}+l^2k^2\mathbf{C}^{(0)}||$; $\mathbf{U}_0$=const - undisturbed solution of matrix equation (15); $\mathbf{A}^{(0)}=\mathbf{A}(\mathbf{U})$ by $\mathbf{U}=\mathbf{U}_0$; $(\nabla\mathbf{B}^{(0)})_{j,k}=(\partial\mathbf{B}_j/\partial\mathbf{U}_k)$ by $\mathbf{U}=\mathbf{U}_0$; $\mathbf{U}\sim\exp i(kx-\omega t)$; $v_g=\partial\omega/\partial k$- the group wave velocity. Here ξ,τ are the "slow-acting" ("compressed") variables introduced by Gardner-Morikawa procedure (Gardner C.S., Greene J.M., Kruskal M.D., Miura R.M., 1967; Gardner C.S., Morikawa G.M., 1969). Taking into accout all above-mentioned it is possible to write for the fundamental harmonic the following equation (Whitham G.B., 1974): $i\partial\varphi/\partial t+0,5(\partial^2\omega/\partial k^2)\partial^2\varphi/\partial\xi^2+\mu|\varphi|^2\varphi-\delta\varphi=0.$ (10)
There is taken an assumption that the fundamental harmonic in considered time interval is dominant and mode's interaction can be neglected. The coefficients in equation (10) are:
$\mu=C/|\mathbf{LR}|$; $C=C_A+C_B+C_C$; $C_A=ik\mathbf{L}\{2(\nabla\mathbf{A}^{(0)}\mathbf{R}^*)\mathbf{R}_2^{(2)}-(\nabla\mathbf{A}^{(0)}\mathbf{R}_2^{(2)})\mathbf{R}^*+(\nabla\mathbf{A}^{(0)}\mathbf{R}_0^{(2)})\mathbf{R}+(\nabla\nabla\mathbf{A}^{(0)}:\mathbf{RR}^*)\mathbf{R}-0,5(\nabla\nabla\mathbf{A}^{(0)}:\mathbf{RR})\mathbf{R}^*\}$; $C_B=\mathbf{L}\{(\nabla\nabla\mathbf{B}^{(0)}(\mathbf{RR}_0^{(2)}+\mathbf{R}^*\mathbf{R}_2^{(2)})+0,5\nabla\nabla\nabla\mathbf{B}^{(0)}:\mathbf{RR}^*\mathbf{R}\}$;
$C_C=-k^2\mathbf{L}\{(\nabla\mathbf{C}^{(0)}\mathbf{R}_2^{(2)})\mathbf{R}+(\nabla\mathbf{C}^{(0)}\mathbf{R}_0^{(2)})\mathbf{R}+(\nabla\nabla\mathbf{C}^{(0)}:\mathbf{R}^*\mathbf{R})\mathbf{R}+0,5(\nabla\nabla\mathbf{C}^{(0)}:\mathbf{RR})\mathbf{R}^*+4(\nabla\mathbf{C}^{(0)}\mathbf{R}^*)\mathbf{R}_2^{(2)}\}$;
$\delta=-d/|\mathbf{LR}|$; $d=\mathbf{L}\{ik\mathbf{A}'^{(0)}+\nabla\mathbf{B}'^{(0)}-k^2\mathbf{C}'^{(0)}\}$; $\mathbf{R}_0^{(2)}=-\{ik[(\nabla\mathbf{A}^{(0)}\mathbf{R}^*)\mathbf{R}+c.c.]+0,5(\nabla\nabla\mathbf{B}^{(0)}\mathbf{RR}^*+c.c.)-k^2[(\nabla\mathbf{C}^{(0)}\mathbf{R}^*)\mathbf{R}+c.c.]\}/||\mathbf{W}_0||$;
$\mathbf{R}_2^{(2)}=-\{ik(\nabla\mathbf{A}^{(0)}\mathbf{R})\mathbf{R}+0,5\nabla\nabla\mathbf{B}^{(0)}\mathbf{RR}k^2(\nabla\mathbf{C}^{(0)}\mathbf{R})\mathbf{R}\}/||\mathbf{W}_2||$; $\nabla\mathbf{A}^{(0)}\mathbf{U}^{(1)}=\mathbf{U}_j^{(1)}(\partial\mathbf{A}/\partial\mathbf{U}_j)$ by $\mathbf{U}=\mathbf{U}_0$;
$\nabla\nabla\mathbf{A}^{(0)}\mathbf{U}^{(1)}\mathbf{U}^{(1)}=\mathbf{U}_j^{(1)}\mathbf{U}_k^{(1)}(\partial^2\mathbf{A}/\partial\mathbf{U}_j\partial\mathbf{U}_k)$ by $\mathbf{U}=\mathbf{U}_0$; $||\mathbf{W}_0||=\det\mathbf{W}_0$; $||\mathbf{W}_2||=\det\mathbf{W}_2$;
$\nabla\nabla\nabla\mathbf{A}^{(0)}\mathbf{U}^{(1)}\mathbf{U}^{(1)}\mathbf{U}^{(1)}=\mathbf{U}_j^{(1)}\mathbf{U}_k^{(1)}\mathbf{U}_m^{(1)}(\partial^3\mathbf{A}/\partial\mathbf{U}_j\partial\mathbf{U}_k\partial\mathbf{U}_m)$ by $\mathbf{U}=\mathbf{U}_0$; c.c.- complex-conjugated.

Here are: $\delta=\delta_r+i\delta_i$; $\mu=\mu_r+i\mu_i$; the complex-conjugated values are sighned with star. For the **A**, **B**, **C**, ω the asymptotic decompositions by order of ε^2 were used. Thus, a rapid wave $\exp i(kx-\omega t)$ has an amplitude multiplier satisfying the non-linear Schrödinger equation (10) with dissipation, which has soliton-like solutions. The solution of the equation (10) for each harmonic having its own equation according to ω (second term in (10)) can be found in the form $\varphi=\psi\exp i(\theta+k_b\xi)$, where $2\pi/k_b$ is a wave length for the amplitude-solution (big wave). Then the following solution is obtained by $\mu_i\neq 0$:
$\theta^{(j)}=\{\mu_r^{(j)}[\psi_\tau^{(j)}]^2-0,5(\partial^2\omega^{(j)}/\partial k^2)_k^{(j)}(k^{(j)})^2+\delta_r^{(j)}\}\tau+2\pi n$; $k^{(j)}=k_b^{(j)}$; $[\psi_\tau^{(j)}]^2=|\delta_i/\mu_i|[1\pm\exp(-2\delta_i\tau)]^{-1}$;
$\psi<\sqrt{|\delta_i/\mu_i|}(+)$, $\psi>\sqrt{|\delta_i/\mu_i|}(-)$; $n\in N$.

The obtained approximate expression allows analyzing stability of excited non-linear waves. Instability is available by $\delta_i>0$, by $\delta_i>0$ the film is stable. But in a non-linear case critical value must be estimated because these stability conditions are only necessary, and not sufficient ones. For the stability of the excited soliton-like waves the critical value of an excitation should be exceeded. In the case considered, this critical value is $\psi_{cr}=\sqrt{|\delta_i/\mu_i|}$. By $\delta_i<0$, $\mu_i<0$, the stability of a soliton-like wave depends on an initial amplitude ψ^2 ($\tau\to-\infty$). For the $(q,\zeta)=R[\varphi]\exp i(kx-\omega t)+c.c.$ there is $q^2=\varphi^2\exp 2i(kx-\omega t)+2|\varphi|^2+c.c.$ Then in case of $\alpha_u=0$ (nonmoistend surface) it is obtained dispersive appropriate correlation:

$$(\omega-k)^2+3i/Rek^2(\omega-k)+\{k^4(\omega-k)^3[(\omega-k)^2+4k^4|\varphi|^2]\}/\{We[2k^4|\varphi|^2-(\omega-k)^2]^{5/2}\}+2iAlkh\partial h/\partial x-k^2/Fr^2=0,\ \omega_m=\pm k_m(im/Re_m-1). \quad (11)$$

For $\sqrt{2}k^2|\varphi|<<|\omega-k|$, the correlation (18) is simplified:

$$(\omega-k)^2+3i/Rek^2(\omega-k)-k^4/We[1+9k^4|\varphi|^2/(\omega-k)^2+30k^8|\varphi|^4/(\omega-k)^4]+2iAlkh\partial h/\partial x-k^2/Fr^2=0.$$

For the $\xi=\varepsilon(x-v_{gj}t)$, $\tau=\varepsilon^2 t$, from equation (16) yields

$$i\partial\varphi^{(j)}/\partial\tau^{(j)}+0{,}5\partial v_{gj}/\partial k\partial^2\varphi^{(j)}/\partial\xi^{(j)2}+\partial\omega_j/\partial|\varphi^{(j)}|^2|\varphi^{(j)}|^2\varphi^{(j)}-\delta^{(j)}\varphi^{(j)}=0.$$

The critical values for the non-linear wave excitation are:

$$|\psi^{(j)}{}_{cr}|=[2We+Fr^2(k_b{}^{(j)})^2]\{Re^2[2We+Fr^2(k_b{}^{(j)})^2]^2+9We^2Fr^2(k_b{}^{(j)})^2\}^{1/2}/[6\sqrt{2}ReFr^2We^{3/2}(k_b{}^{(j)})^2];$$

$$q^{(j)}{}_{cr}=[2We+Fr^2(k_b{}^{(j)})^2]\{We[4(1/Fr+0{,}5Fr(k_b{}^{(j)})^2/We)^2+9(k_b{}^{(j)})^2/Re^2]\}^{1/2}/[6\sqrt{2}FrWe(k_b{}^{(j)})^2];$$

$\zeta^{(j)}{}_{cr}=q^{(j)}{}_{cr}[k_b{}^{(j)}/(\omega_j-k_b{}^{(j)})]$. For $Re>>1$ the dissipation effects are negligibly small, therefore the non-linear Schrödinger equation (10) has solitary solutions when $\varphi^{(j)}\to 0$ by $|\xi^{(j)}|\to\infty$:

$$\varphi^{(j)}=[-2A^{(j)}/\mu_r{}^{(j)}]^{1/2}\mathrm{sech}\{[-0{,}5(\partial^2\omega_j/\partial k^2)/A^{(j)}]^{-1/2}\xi^{(j)}\}\exp(-iA^{(j)}\tau^{(j)}),\quad A^{(j)}=-0{,}5\mu_r{}^{(j)}[\varphi_0{}^{(j)}]^2,$$

where $\varphi_0{}^{(j)}$ is the value of $\varphi^{(j)}$ by $\tau=0$, $\xi^{(j)}=0$. The solitary wave's width is $\Delta_s{}^{(j)}=[(\partial^2\omega_j/\partial k^2)/(\mu_r{}^{(j)}\varphi_0{}^{(j)})]^{1/2}$,

where $A_s{}^{(j)}=\varphi_0{}^{(j)}$ is the wave's amplitude and $v_s{}^{(j)}=\partial^2\omega_j/\partial k^2$ is its velocity. The non-linear addend to the solitary wave's frequency is the next $\Delta\omega_s=0{,}5\mu_r{}^{(j)}(\varphi_0{}^{(j)})^2$. An electromagnetic modulation in a linear approach gives:

$$\omega=k-1.5ik^2/Re\pm\{k^2(1/Fr^2-,25k^2/Re^2+k^2/We)+2ik\bullet k_mAl[h_0+2h_m\cos(k_mx-\omega_mt)][2h_m\sin(k_mx-\omega_mt)]\}^{1/2}.$$

Moreover, the soliton-like excitation for a film flow requests also the Lighthill's condition:

$$We(k+1)(1+k^2Fr^2/We)>k^4Fr^2, \quad (12)$$

that is a limitation on minus-plus signes of the coefficients by a terms $\partial^2\varphi/\partial\xi^2$ and a non-linear term of a standard evolutionary equation. As follows from (12), this criteria be easier to satisfy by small value of k. The calculations showed the next critical parameters:

$|\psi_s|_{cr}=3*10^{-2}$ - in general case; $|\psi_s|_{cr}=3*10^{-3}$ - by $k>>1$ (short-wave solitons);
$|\psi_s|_{cr}=10^{-6}$-10^{-8} - by $k<<1$ (long-wave solitons).

A number of different linear, as well as non-linear modelling situations were considered. As a result there were obtained some interesting regularities of parametric wave excitation and suppression in film flows including the three new phenomena of parametric film decay: resonance decay, soliton-like decay and shock-wave decay. Based on these new phenomena, the prospective dipergators and granulators were developed, created and tested for some metals and other materials.

Conclusions

The new phenomena on parametric control in film flows and channel flows with solidifying on a wall were revealed. Except theoretical importance, they were proven as highly important for industrial and other applications.

References

1. Kazachkov I.V. Electromagnetic wave excitation and suppression of films, Magnetohydrodynamics, vol.32, no.1; Jan.-March 1996; p.68-73.

2. Kazachkov I.V. Analytical solutions for electromagnetic excitation of nonlinear waves in films, Magnetohydrodynamics, vol.27, no.2; April-June 1991; p.190.

3. Kazachkov I.V., Kolesnichenko A.F. About the phenomena of electromagnetic excitation of film decay, Pis'ma v Zhurnal Tekhnicheskoi Fiziki (Letters to the Journal of Techn. Phys.), 1991, Vol.17, No 3, pp. 40-43 (In Russian).